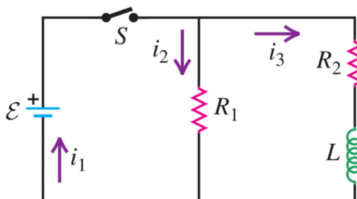


## Physics 1C Discussion (Week 4)

### Problem 1

•• **CALC** An inductor with inductance  $L = 0.300$  H and negligible resistance is connected to a battery, a switch  $S$ , and two resistors,  $R_1 = 12.0\ \Omega$  and  $R_2 = 16.0\ \Omega$  (Fig. P30.50). The battery has emf  $96.0$  V and negligible internal resistance.  $S$  is closed at  $t = 0$ . (a) What are the currents  $i_1$ ,  $i_2$ , and  $i_3$  just after  $S$  is closed? (b) What are  $i_1$ ,  $i_2$ , and  $i_3$  after  $S$  has been closed a long time? (c) What is the value of  $t$  for which  $i_3$  has half of the final value that you calculated in part (b)? (d) When  $i_3$  has half of its final value, what are  $i_1$  and  $i_2$ ?

**Figure P30.50**



#### Part a)

Just after  $S$  is closed, the inductor,  $L$ , resists the change in the current (and thereby change in magnetic field). Due to this, current is not able to instantly flow through  $L$ , and  $i_3 = 0$ . To see this mathematically,

$$i_3(t) = \frac{\mathcal{E}}{R_2}(1 - e^{-(R_2/L)t}), \quad (1)$$

for an  $R$ - $L$  circuit. At  $t = 0$ , this is just zero. So,  $i_3 = 0$ . Then, using Kirchhoff's voltage law,

$$\mathcal{E} = i_2 R_1 \implies i_2 = \mathcal{E}/R_1 = 8.0\text{A} \quad (2)$$

From Kirchhoff's current law, then,

$$i_1 = i_2 + i_3 \implies i_1 = i_2 = 8.0\text{A} \quad (3)$$

#### Part b)

After a long time, the current through the inductor is constant. To see this mathematically,

$$\lim_{t \rightarrow \infty} i_3(t) = \lim_{t \rightarrow \infty} \frac{\mathcal{E}}{R_2}(1 - e^{-(R_2/L)t}) = \frac{\mathcal{E}}{R_2} \quad (4)$$

Since the inductor does not matter, we effectively have two resistors in series. As such, from Kirchhoff's voltage law,

$$\mathcal{E} = i_2 R_1 \implies i_2 = \frac{\mathcal{E}}{R_1} = 8.0\text{A} \quad (5)$$

$$\mathcal{E} = i_3 R_2 \implies i_3 = \frac{\mathcal{E}}{R_2} = 6.0\text{A} \quad (6)$$

From the current law,

$$i_1 = i_2 + i_3 = \mathcal{E} \left( \frac{1}{R_1} + \frac{1}{R_2} \right) = 14.0\text{A} \quad (7)$$

**Part c)**

We are looking for  $t_0$  such that  $i_3(t_0) = 3.0\text{A}$ . Plugging in our formula for the current of a  $RL$  circuit,

$$i_3(t_0) = \frac{\mathcal{E}}{R_2} \left( 1 - e^{-(R_2/L)t_0} \right) \quad (8)$$

Rearranging to solve for  $t_0$ ,

$$1 - \frac{i_3(t_0)R_2}{\mathcal{E}} = e^{-(R_2/L)t_0} \quad (9)$$

Taking the natural log of both sides,

$$\ln \left( 1 - \frac{i_3(t_0)R_2}{\mathcal{E}} \right) = -(R_2/L)t_0 \quad (10)$$

So,

$$t_0 = -\frac{L}{R_2} \ln \left( 1 - \frac{i_3(t_0)R_2}{\mathcal{E}} \right) = 13.0 \times 10^{-3}\text{s} \quad (11)$$

**Part d)**

Using Kirchhoff's voltage law,

$$\mathcal{E} = i_2 R_1 \implies i_2 = \frac{\mathcal{E}}{R_1} = 8.0\text{A}. \quad (12)$$

Finally, from Kirchhoff's current law,

$$i_1 = i_2 + i_3 = 11.0\text{A} \quad (13)$$